The “natural” language of mathematics and its technical challenges in the TLS

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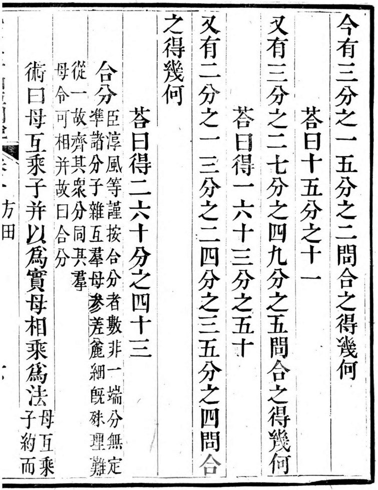
24 November, 2022

The language of mathematics in China is natural:

- discursive sections: paratext and in general in meta-mathematical commentaries
- problems and procedures are versified (from Song dynasty on)
- constant borrowing of common words for specific mathematical concepts


今有三分之一五分之二問合之得幾何
荅曰十五分之十一
又有三分之二七分之四九分之五問合之得幾何
荅曰得一六十三分之五十
又有二分之一三分之二四分之三五分之四問合之得幾何
荅曰得二六十分之四十三
合分臣淳風等謹按合分者數非一端分無定準諸分子雜互羣母參差麄細既殊理難從一故齊其衆分同其羣母令可相并故曰合分
術曰母互乘子并以爲實母相乘爲法母互乘子約而
方田

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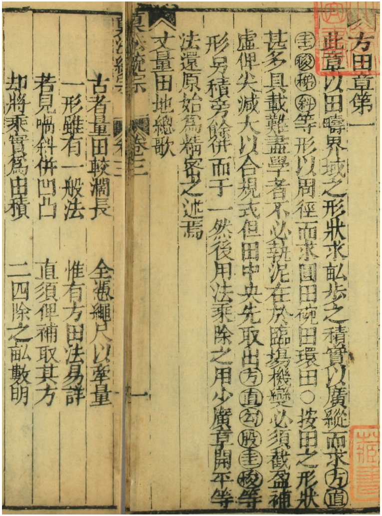
算法統宗 卷三 一

方田章第一

此章以田疇界域之形狀求畝步之積實以廣縱而求(方)(直)(圭)(梭)(梯)(斜)等形以周徑而求圓田碗田環田○按田之形狀甚多具載難盡學者不必執泥在於臨場機變必須截盈補虛俾尖減大以合規式但田中央先取出(方)(直)(勾)(股)(圭)(梭)等形另積旁餘併而于一然後用法乘除之用少廣商開平等法還原始爲精密之述焉

丈量田地總歌

古者量田較濶長　全憑繩尺以牽量
一形雖有一般法　惟有方田法易詳
若見喎斜併凹凸　直須俾補取其方
却將乘實爲田積　二四除之畝數明

算法統宗 卷三

Nine Chapters on Mathematical Procedures 九章算術 (1st cent. AD)

合分術曰：

母互乘子,

并以為實。

母相乘為法。

Nine Chapters on Mathematical Procedures 九章算術 (1st cent. AD)

合分術曰：	The art of uniting separate parts prescribes:
母互乘子,	“Mothers and Sons, mutually mount each other,
并以為實。	come together for your fruit,
母相乘為法。	take mothers who mount each other as model!”

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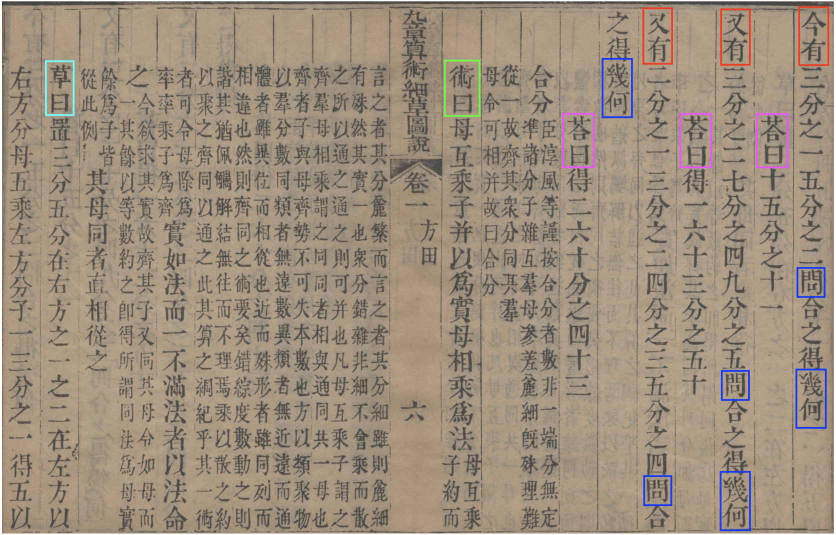
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九章算術細草圖說 卷一 方田 六

言之者其分麤繁而言之者其分細雖則麤細有殊然其實一也衆分錯雜非細不會乘而散之所以通之通之則可并也凡母互乘子謂之齊羣母相乘謂之同同者相與通同共一母也齊者子與母齊勢不可失本數也方以類聚物以羣分數同類者無遠數異類者無近遠而通體者雖異位而相從也近而殊形者雖同列而相違也然則齊同之術要矣錯綜度數動之則諧其猶佩觿解結無往而不理焉乘以散之約以聚之齊同以通之此其算之綱紀乎其一術者可令母除爲率率乘子爲齊實如法而一不滿法者以法命之今欲求其實故齊其子又同其母令如母而一其餘以等數約之卽得所謂同法爲母實餘爲子皆從此例其母同者直相從之
草曰置三分五分在右方之一之二在左方以
右方分母五乘左方分子一三分之一得五以

Every fully-fledged mathematical problem can be divided into self-contained segments. These segments are canonised in the *Nine Chapters*' scheme. They are sharply demarcated by a series of linguistic markers:

- Assumptions about the givens: 今有
- Question 問...幾何
- Answer 答曰
- General procedure to solve the problem, or an entire class of problems 術曰
- The (concrete) calculation sketch 草曰, i.e. the algorithm (from the Song dynasty on)

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The nine returns (*Jiugui* 九歸)

三一三十一
三二六十二
逢三進一十

Three-one : thirty-one.

Three-two : Sixty-two.

When one encounters a three, advance one ten.

七一下加三
七二下加六
七三四十二
七四五十五
七五七十一
七六八十四
逢七進一十

Seven-one: add three to the next.

Seven-two: add six to the next.

Seven-three: fourty-two.

Seven-four: fifty-five.

Seven-five seventy-one.

Seven-six: eighty-four.

When one encounters a seven, advance one ten.

jiao cao 茭草

$$1\cdot1+1\cdot2+1\cdot3+1\cdot4+\ldots+1\cdot n$$

luo yi 落一

$$\mathbf{n}\cdot1+(\mathbf{n}-\mathbf{1})\cdot2+(\mathbf{n}-\mathbf{2})\cdot3+(\mathbf{n}-\mathbf{3})\cdot4+\ldots+\mathbf{1}\cdot n$$

san jiao 三角

$$=1\cdot1+1\cdot(1+2)+1\cdot(1+2+3)+\ldots+1\cdot(1+2+\ldots+n)$$

si jiao 四角

$$1\cdot1+2\cdot2+3\cdot3+4\cdot4+\ldots+\mathbf{n}\cdot n$$

san jiao luo yi 三角落一

$$\mathbf{n}\cdot1+(\mathbf{n}-\mathbf{1})\cdot(1+2)+(\mathbf{n}-\mathbf{2})\cdot(1+2+3)+\ldots+\mathbf{1}\cdot(1+2+\ldots+n)$$

sa xing 撒星

$$=(1+2+\ldots+n)\cdot1+(1+\ldots+n-1)\cdot2+(1+\ldots+n-2)\cdot3+\ldots+1\cdot n$$

$$1\cdot1+\mathbf{2}\cdot(1+2)+\mathbf{3}\cdot(1+2+3)+\ldots+\mathbf{n}\cdot(1+2+\ldots+n)$$

lan feng 嵐峰

$$=(1+2+\ldots+n)\cdot1+(2+3+\ldots+n)\cdot2+(3+\ldots+n)\cdot3+\ldots+n\cdot n$$

si jiao luo yi 四角落一

$$\mathbf{n}\cdot1\cdot1+(\mathbf{n}-\mathbf{1})\cdot2\cdot2+(\mathbf{n}-\mathbf{2})\cdot3\cdot3+(\mathbf{n}-\mathbf{3})\cdot4\cdot4+\ldots+\mathbf{1}\cdot n\cdot n$$

sa xing geng luo yi 撒星更落一

$$\mathbf{1}\cdot(1+2+\ldots+n)+(\mathbf{1}+\mathbf{2})\cdot(1+\ldots+n-1)+(\mathbf{1}+\mathbf{2}+\mathbf{3})\cdot(1+\ldots+n-2)+\ldots+(\mathbf{1}+\ldots+\mathbf{n})\cdot1$$

san jiao sa xing 三角撒星

$$=\mathbf{n}\cdot1+(\mathbf{n}-\mathbf{1})\cdot[1+(1+2)]+(\mathbf{n}-\mathbf{2})\cdot[1+(1+2)+(1+2+3)]+\ldots+\mathbf{1}\cdot[1+(1+2)+\ldots+(1+2+\ldots+n)]$$

lan feng geng luo yi 嵐峰更落一

$$(1+2+\ldots+n)\cdot1+(2+3+\ldots+n)\cdot(1+2)+(3+\ldots+n)\cdot(1+2+3)+\ldots+n\cdot(1+\ldots+n)$$

san jiao lan feng 三角嵐峰

$$=\mathbf{1}\cdot1+\mathbf{2}\cdot(1+(1+2))+\mathbf{3}\cdot(1+(1+2)+(1+2+3))+\ldots+\mathbf{n}\cdot(1+(1+2)+\ldots+(1+2\ldots+n))$$

si jiao lan feng 四角嵐峰

$$1\cdot1+2\cdot(1\cdot1+2\cdot2)+3\cdot(1\cdot1+2\cdot2+3\cdot3)+\ldots+n\cdot(1\cdot1+\ldots+n\cdot n)$$

san jiao sa xing geng luo yi 三角撒星更落一

$$(\mathbf{1}+\mathbf{2}+\ldots+\mathbf{n})\cdot1+(\mathbf{1}+\mathbf{2}+\ldots+\mathbf{n}-\mathbf{1})\cdot[1+(1+2)]+(\mathbf{1}+\ldots+\mathbf{n}-\mathbf{2})\cdot[1+(1+2)+(1+2+3)]+\ldots+\mathbf{1}\cdot[1+\ldots+(1+\ldots+n)]$$

Kapitel 4 - Abbildung der Akkumulationshierarchie

Li Shanlan 李善蘭. *Analogical categories of discrete accumulations* (*Duoji bilei* 垛積比類), 1867.

Li Shanlan, *Duoji bilei* 垛積比類 (1867)

$$\sum_{k=1}^{n}\binom{k}{1}^2 = 1\cdot\binom{n+2}{3} + 1\cdot\binom{n+1}{3}$$

$$\sum_{k=2}^{n+1}\binom{k}{2}^2 = 1\cdot\binom{n+4}{5} + 4\cdot\binom{n+3}{5} + 1\cdot\binom{n+2}{5}$$

$$\sum_{k=3}^{n+2}\binom{k}{3}^2 = 1\cdot\binom{n+6}{7} + 9\cdot\binom{n+5}{7} + 9\cdot\binom{n+4}{7} + 1\cdot\binom{n+3}{7}$$

$$\sum_{k=4}^{n+3}\binom{k}{4}^2 = 1\cdot\binom{n+8}{9} + 16\cdot\binom{n+7}{9} + 36\cdot\binom{n+6}{9} + 16\cdot\binom{n+5}{9} + 1\cdot\binom{n+4}{9}$$

$$\sum_{k=j}^{n+j-1}\binom{k}{j}^2 = ?$$

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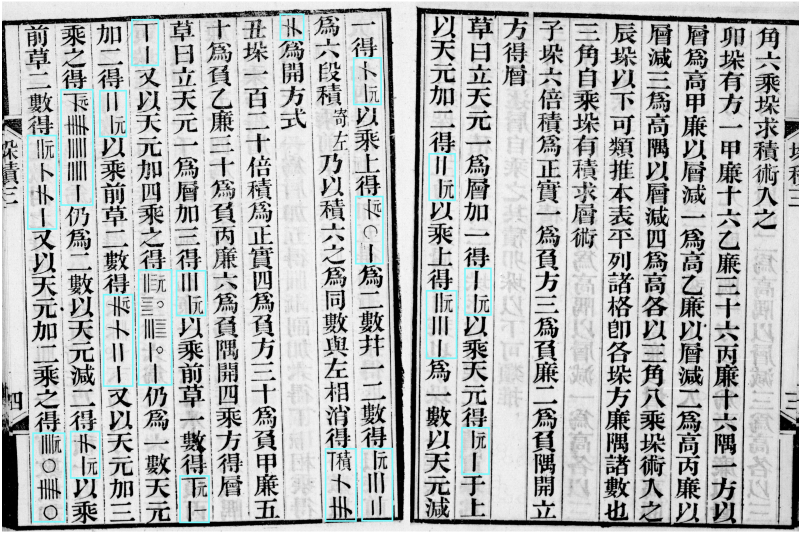

角六乘垛求積術入之
卯垛有方一甲廉十六乙廉三十六丙廉十六隅一方以
層爲高甲廉以層減一爲高乙廉以層減二爲高丙廉以
層減三爲高隅以層減四爲高各以三角八乘垛術入之
辰垛以下可類推本表平列諸格卽各垛方廉隅諸數也
三角自乘垛有積求層術
子垛六倍積爲正實一爲負方三爲負廉二爲負隅開立
方得層
草曰立天元一爲層加一得[illegible]以乘天元得[illegible]于上
以天元加二得[illegible]以乘上得[illegible]爲一數以天元減
一得[illegible]以乘上得[illegible]爲二數幷一二數得[illegible]
爲六段積寄左乃以積六之爲同數與左相消得[illegible]
[illegible]爲開方式
丑垛一百二十倍積爲正實四爲負方三十爲負甲廉五
十爲負乙廉三十爲負丙廉六爲負隅開四乘方得層
草曰立天元一爲層加三得[illegible]以乘前草一數得[illegible]
[illegible]又以天元加四乘之得[illegible]仍爲一數天元
加二得[illegible]以乘前草二數得[illegible]又以天元加三
乘之得[illegible]仍爲二數以天元減二得[illegible]以乘
前草二數得[illegible]又以天元加二乘之得[illegible]

Chinese Text Project

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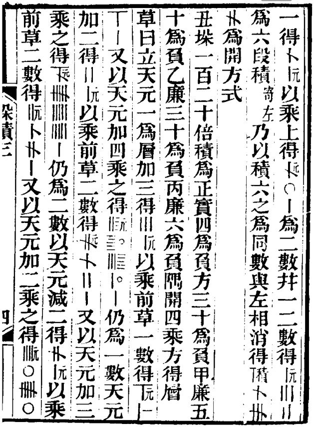

一得卜阮以乘上得挺何斛斛數井且一數得玩川川
糊八殷積青左乃以積八之為同數與左相消得附木拂
林為開方式
口媒一百一十倍積赫正實泗為貧方一十為負甲廉五
十為負乙屬臘十為負丙廉六為負隅開四乘方得層
草日立天元一為膺加一得元以乘前草憂得百山
汙川文以天元加四乘之得慚壺腫口仍為藪零兀
理再亢以乘前草一數得稱水口又以天元加一
乘之得竇刪刪刪仍種藪以零冗睡再林元以鹽
前草一數得阮卜林又以天元加叫乘之得脆口刪口

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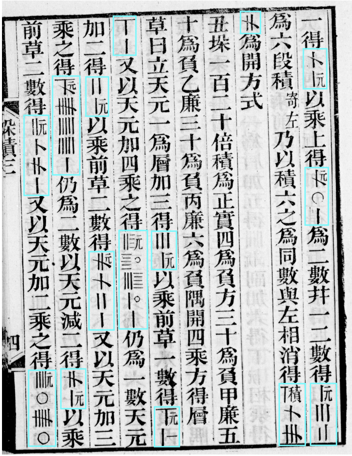
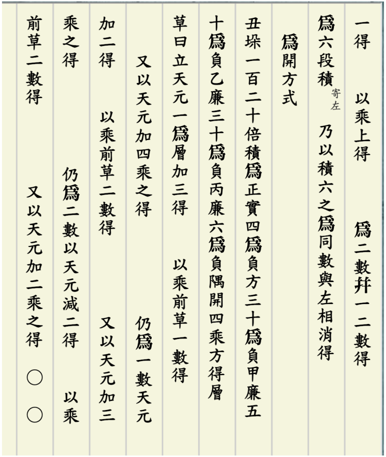




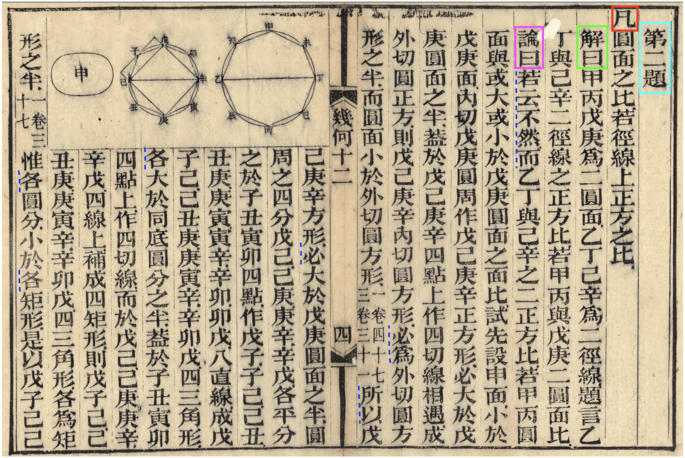
一得　以乘上得　爲二數幷一二數得
爲六段積寄左　乃以積六之爲同數與左相消得
爲開方式
丑垛一百二十倍積爲正實四爲負方三十爲負甲廉五
十爲負乙廉三十爲負丙廉六爲負隅開四乘方得層
草曰立天元一爲層加三得　以乘前草一數得
又以天元加四乘之得　仍爲一數天元
加二得　以乘前草二數得　又以天元加三
乘之得　仍爲二數以天元減二得　以乘
前草二數得　又以天元加二乘之得　○　○

Digital version of the *Sequel to the Siku quanshu* 續修四庫全書
Author: 凱希多媒體公司, 大鐸資訊股份有限公司, 得泓資訊有限公司

Series title: 雕龍–中國日本古籍全文檢索資料庫

Goal of our research: identify stylistic codes and their historical evolution:

- philosophical/conceptual
- procedural/prescriptive
- algorithmic/numeric (from Song dynasty onwards)
- logical/argumentative (influence of translations of Western mathematical works?)


第二題
凡圓面之比若徑線上正方之比
解曰甲丙戊庚爲二圓面乙丁己辛爲二徑線題言乙
丁與己辛二徑線之正方比若甲丙與戊庚二圓面比
論曰若云不然而乙丁與己辛之二正方比若甲丙圓
面與或大或小於戊庚圓面之面比試先設申面小於
戊庚面內切戊庚圓周作戊己庚辛正方形必大於戊
庚圓面之半蓋於戊己庚辛四點上作四切線相遇成
外切圓正方則戊己庚辛內切圓方形必爲外切圓方
形之半而圓面小於外切圓方形（一卷四十七 三卷三十一）所以戊

幾何十二　四

己庚辛方形必大於戊庚圓面之半圓
周之四分戊己己庚庚辛辛戊各平分
之於子丑寅卯四點作戊子子己己丑
丑庚庚寅寅辛辛卯卯戊八直線成戊
子己己丑庚庚寅辛辛卯戊四三角形
各大於同底圓分之半蓋於子丑寅卯
四點上作四切線而於戊己己庚庚辛
辛戊四線上補成四矩形則戊子己己
丑庚庚寅辛辛卯戊四三角形各爲矩
形之半（一卷三十七）惟各圓分小於各矩形是以戊子己己

Translation of Euclid's *Elements* 幾何原本
by Li Shanlan 李善蘭 and Alexander Wylie (1859)